Kuwait University | Math 101 | Date: | May 4, 2006
Dept. of Math. & Comp. Sci. | Second Exam | Duration: | 90 minutes

Calculators, mobile phones, pagers and all other mobile communication equipment are not allowed

Answer the following questions:

1. Find $f''(1)$, where $f(x) = \dfrac{x^2-1}{x}$. (2 pts.)

2. Use differentials to approximate: $1 + (8.01)^{\frac{2}{3}}$. (3 pts.)

3. Find an equation of the normal line at $x = 1$, to the graph of
$$x^2y + \sin(xy - y) = 2.$$ (4 pts.)

4. (a) State Rolle's Theorem. (1 pt.)

   (b) Use Rolle's Theorem to show that the graph of $f(x) = x^4 + x^3 - 3x^2 + x + 1$, cannot have more than one inflection point in $[0,1]$. (3 pts.)

5. The radius of a closed right circular cylinder is decreasing at a rate of 1 cm/sec and the height is increasing at a rate of 4 cm/sec. Find the rate at which the total surface area of the cylinder is changing when the radius is 6 cm and the height is 10 cm.

   (4 pts.)

6. Let $f(x) = \dfrac{2x^2-2x+1}{x^2}$, and given that $f'(x) = \dfrac{2x-2}{x^3}$ and $f''(x) = \dfrac{6-4x}{x^4}$.

   (a) Find the vertical and horizontal asymptotes for the graph of f, if any.

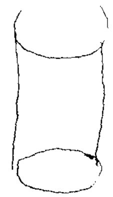
   (b) Find the intervals on which f is increasing and the intervals on which f is decreasing. Find the local extrema of f, if any.

   (c) Find the intervals on which the graph of f is concave upward and the intervals on which the graph of f is concave downward. Find the points of inflection, if any.

   (d) Sketch the graph of f. (8 pts.)

Q5

$S =$ Surface Area of right circular cylinder $= 2\pi r^2 + 2\pi r h = S$, $\frac{dr}{dt} = -1$ cm/sec, $\frac{dh}{dt} = 4$ cm/sec. Find $\frac{ds}{dt}$. $r = 6, h = 10$



$\therefore \frac{ds}{dt} = 4\pi r \frac{dr}{dt} + 2\pi\left[r\frac{dh}{dt} + h\frac{dr}{dt}\right]$

$= 4\pi(6)(-1) + 2\pi[6\times 4 - 10\times 1] = -24\pi + 2\pi(24-10)$

$\frac{ds}{dt} = -24\pi + 28\pi = 4\pi \text{ cm}^2/\text{sec}$

Q6 x-Intercept $y = 0 \Rightarrow 2x^2 - 2x + 1 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4-8}}{2}$

no x-intercept.

y-intercept $x = 0$, $y = \infty$, no y-intercept

V.A $\Rightarrow y = \lim_{x\to 0} \frac{2x^2-2x+1}{x^2} = \infty$ $\therefore x = 0$ is V.A

H.A $\rightarrow y = \lim_{x\to\infty} \frac{2x^2-2x+1}{x^2} = 2$ $\quad y = 2$ H.A

Now put $y = 2$ in $y = \frac{2x^2-2x+1}{x^2} \Rightarrow 2x^2 = 2x^2 - 2x + 1 \Rightarrow x = \frac{1}{2}$

So graph intersects H.A at $(\frac{1}{2}, 2)$

$f'(x) = \frac{2(x-1)}{x^3}$

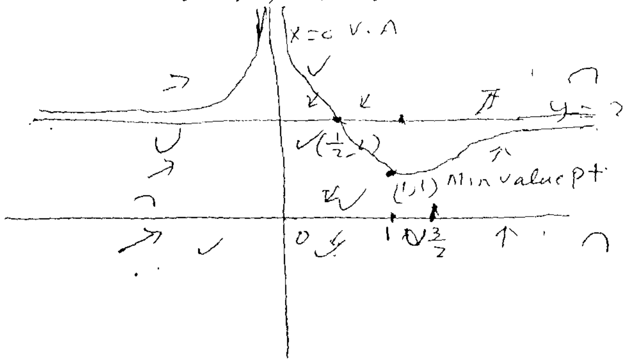
	0	1		
$- - -$	$-$	$+ + +$	$x-1$
$- - -$	$+$	$+ + +$	x^3
$+$	$-$	$+$	

$f \uparrow (-\infty, 0) \cup (1, \infty), \downarrow (0, 1)$ $\therefore x = 1$ gives min value, $y = 1$ $(1,1)$ min value pt

$f''(x) = \frac{6-4x}{x^4}$

	0	3/2		
$+ + +$	$+$	$- - -$	$6-4x$
$+ + +$	$+$	$+ + +$	x^4
$+$	$+$	$-$	

$\Rightarrow$ Concave up $(-\infty, 0) \cup (0, \frac{3}{2})$ Concave down $(\frac{3}{2}, \infty)$



Ans

[illegible]

$x - \frac{1}{x} = x - x^{-1}$, $f'(x) = 1 + x^{-2}$, $f''(x) = -\frac{2}{x^3}$

$f''(1) = -\frac{2}{1} = -2$ Ans

Q2 Let $y = 1 + x^{2/3}$ $\quad x = 8 \quad dx = .01$

$y = 1 + 8^{2/3} = 1 + (2^3)^{2/3} = 1 + 4 = 5$.

$dy = \frac{2}{3}x^{-\frac{1}{3}}dx = \frac{2}{3x^{1/3}}dx = \frac{2}{3\times 2}(.01) = \frac{.01}{3} = .003$

$\therefore y + dy = 5 + .0033 = 5.0033$ Ans

2nd Method $f(8.01) \approx f(8) + f'(8)\Delta x$

$\approx 5 + \frac{1}{3}(.01) \approx 5.0033$.

Q3 $x^2 y + \sin(xy - y) = 2$ $\quad x = 1, \ y = 2$.

$\therefore 2xy + x^2y' + \cos(xy-y)[xy' + y - y'] = 0$

$\therefore 4 + y' + \cos 0\,(y' + 2 - y') = 0 \Rightarrow 4 + y' + 2 = 0$

$y' = -6 = m_1 =$ slope of Tangent

Slope of normal $m_2 = -\frac{1}{m_1} = \frac{1}{6}$.

Eq of normal $y - y_1 = m_2(x - x_1)$

$= y - 2 = \frac{1}{6}(x - 1) \Rightarrow 6y - 12 = x - 1$

$\Rightarrow x - 6y + 11 = 0$ Ans

Q4 (a) Statement of Rolle's theorem (see book)

b) let $f(x)$ has Two Pts of inflection in $(0,1)$ say 'a' and 'b'

$\therefore f''(a) = f''(b) = 0$ Now according Rolle's Theorem There must be $c \in (a,b)$ such that $f'''(c) = 0$ but $f'''(x) = 24x + 6 \neq 0$ in $(0,1)$

$\therefore$ contradiction, Hence $f(x)$ has one Point of inflection in $(0,1)$.

2nd Method $f''(x) = 12x^2 + 6x - 6 = 0 \Rightarrow 2x^2 + x - 1 = 0$

$(2x-1)(x+1) = 0 \quad x = \frac{1}{2}, -1$

$\therefore f''(\frac{1}{2}) = 0$ in $(0,1)$, $-1 \notin (0,1)$

[illegible]